Unit 4: Related Rates Problems

Name: ___________________

*What is meant by Related Rates?*

Related Rates problems refer to the situation where there are two or more variables that are closely related which are changing with respect to ***time***.

## *Procedure:*

1. Make a DRAWING. Assign variables to the different quantities and label the appropriate parts of the figure with these variables.

2. State what is GIVEN about the variables.
   Determine any rates of change given in the conditions of the problem. (*The derivative of each variable is with respect to time).*

3. Determine an EQUATION, which is appropriate for the drawing and the conditions of the problem.
   Examples:
   a) Pythagorean Theorem
   b) Area Formulas
   c) Surface Area/Volume Formulas

4. If necessary, find values of missing parts in the figure for the moment of in time being discussed.

5. Find the DERIVATIVE of the equation implicitly with respect to time.

6. Substitute all known information into the derivative and SOLVE for the desired rate of change.

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# Related Rates Examples

1. At 1:00 pm, a plane travels over CB West traveling due south at a constant velocity of 450 mph. At 1:30 pm another plane passes over CB West traveling due east at 600 mph. At 3:00 pm, how rapidly will the distance between the planes be increasing?

2. A rocket is launched and travels straight up at a constant velocity of 750 mph. An observer is 2.5 miles away watching the launch. How fast is the distance from the observer to the rocket increasing when the rocket is one mile high?

3. A motorcycle is traveling south toward an intersection at a rate of 45 mph while a truck is traveling west from the intersection at a rate of 50 mph. Find the rate of change on the distance between the two moving vehicles if the motorcycle is still 5 miles north of the intersection and the truck is 3 miles east of the intersection.

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4. Assume that a spherical balloon retains its shape as it is being filled. If air is being pumped in at a rate of $5\,cm^3/s$, how rapidly is the radius changing when the radius is equal to 15 cm? ($V = \frac{4}{3}\pi r^3$)

5. The blueprint for a construction project is being enlarged using specialized digital imaging equipment for an engineer. If the original blueprint being used has a width of 11 in. and a length of 17 in. while the width is being enlarged .5 in/sec and the length is being enlarged 1 in/sec, how is the area of the blueprint changing after 15 seconds? ($A = lw$)

6. A cylindrical pool has a diameter of 30 ft and a height of 5 ft. If the pool is being filled with water at a rate of 20 $ft^3$/min and the level of the water is 3 ft, how fast is the level of water rising at this moment? ($V = \pi r^2 h$)

**7.** A cone shaped storage facility, which is 120 ft. tall and has a radius at the base of 40 ft. is being filled with gasoline at a rate of 60 $ft^3/\min$. How fast is the gasoline level rising when the height of the gasoline is 10 ft.? ($V = \frac{1}{3}\pi r^2 h$)

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# AP Calculus AB
# Unit 4 Worksheet 1

**SHOW all work on a separate sheet of paper. Be sure to include an appropriate drawing.**

1. An airplane flying west at 400 mph passes over Doylestown at 11:30 a.m. A second plane at the same altitude flies over Doylestown at 12:00 noon going south at 500 mph. How fast are the planes separating at 1:00 p.m.?

2. An airplane flying horizontally at an altitude of 1 mile passes directly over an observer. If the plane is traveling at a constant velocity of 240 mph, how fast is the plane's distance from the observer increasing 30 seconds later (*Hint:* 30 seconds = $1/120$ of an hour).

3. A man on a dock is pulling in a rope fastened to the bow of a small boat. If the man's hands are 12 feet higher than the point where the rope is attached to the boat, and if he is pulling the rope in at a rate of 3 feet per second, how fast is the boat approaching the dock when 20 feet of rope still need to be pulled in?

4. A 20-foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the level pavement directly away from the wall at 2 feet per second, how fast is the top of the ladder moving down the wall when the bottom of the ladder is 4 feet from the wall?

5. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child letting out the kite string at the moment that 150 feet of string is out?

6. Two ships sail from the same island port, one going north at 24 knots and the other east at 30 knots. The northbound ship departed at 9:00 a.m. and the eastbound ship left at 11:00 a.m. How fast is the distance between them increasing at 2:00 p.m.?

7. A small balloon is released at a point 150 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 8 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 50 feet high?

8. An airplane is flying at an altitude of 7 miles and passes directly over a radar antenna. When the distance between the plane an the radar antenna is 10 miles, the radar detects that the distance between the antenna and the plane is increasing at a rate of 300 mph. What is the speed of the airplane at that moment?

9. A hiker is 2 miles to the South headed **away** from the campgrounds at a rate of 5 mph. Another hiker is 3 miles to the East of the campgrounds heading **towards** the campgrounds at a rate of 4 mph. How is the distance between them changing after 30 minutes have passed?

10. A rocket rising vertically is tracked by a radar station that is on the ground 5 miles from the launchpad. How fast is the rocket rising when it is 4 miles high and its distance from the radar station is increasing at a rate of 2000 mph?

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# Unit 4 Worksheet 1

11. Each edge of a cube is increasing at a rate of 3 inches per second. How fast is the volume of the cube increasing when an edge of the cube is 10 inches long? $\left(V = e^3\right)$

12. Assuming that a bubble retains its spherical shape as it expands, how fast is its radius increasing when its radius is 2 inches, if air is blown into it at a rate of 4 cubic inches a second? $\left(V = \frac{4}{3}\pi r^3\right)$

13. Oil from a ruptured tanker spreads in a circular pattern. If the radius of the circle increases at a constant rate of 1.5 feet per second, how fast is the enclosed area increasing at the end of 2 hours? $\left(A = \pi r^2\right)$

14. A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 feet per second. How rapidly is the area enclosed by the ripple increasing at the end of 10 seconds? $\left(A = \pi r^2\right)$

15. A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate is air being removed when the radius is 9 cm? $\left(V = \frac{4}{3}\pi r^3\right)$

16. At the start, a triangle has a base of 6 cm and a height of 14 cm. The base is increasing at a rate of 2 cm/sec. and the height is increasing at a rate of 3 cm/sec. How rapidly is the area of the triangle increasing after 10 seconds? $\left(A = \frac{1}{2}bh\right)$

17. A rectangle starts with a length of 15 inches and a width of 8 inches. The length is increasing at a rate of 4 in/min and the width is increasing at a rate of 2 in/min. How rapidly is the area of the rectangle increasing after 25 minutes? $\left(A = lw\right)$

18. A cylindrical oil filter, that has a radius of 6 inches and a height of 12 inches, is slowly being drained by a technician during a recent servicing. If the oil level inside the filter is currently 9 inches and it is draining at a rate of 2 $in^3$/min, how is the height changing at this moment? $((V = \pi r^2 h)$

19. A student is using a straw to drink from a conical paper cup at a rate of 3 cubic centimeters a second. If the height of the cup is 10 centimeters and the diameter at the opening is 6 centimeters, how fast is the level of the liquid falling when the depth is 5 centimeters? $\left(V = \frac{1}{3}\pi r^2 h\right)$

20. Water is pouring into a conical storage tank at the rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of the circular opening is 6 feet, how fast is the water level rising when the water is 4 feet deep? $\left(V = \frac{1}{3}\pi r^2 h\right)$